

# Gravitational particle production in braneworld cosmology

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Gravitational particle production in time variable metric of an expanding universe is efficient only when the Hubble parameter  $H$  is not too small in comparison with the particle mass. In standard cosmology, the huge value of the Planck mass  $M_{Pl}$  makes the mechanism phenomenologically irrelevant. On the other hand, in braneworld cosmology the expansion rate of the early universe can be much faster and many weakly interacting particles can be abundantly created. Cosmological implications are discussed.

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*Introduction* – It is a well known fact that particles can be created by classical backgrounds such as time variable spacetime metrics [1, 2] and oscillating (or any other time-dependent) fields [3]. In particular, gravitational particle production in time varying metric is an inevitable phenomenon which does not depend on particle interactions, because according to General Relativity all the forms of energy couple to gravity with the same strength (Equivalence Principle), so that even very weakly interacting or sterile ones can be abundantly created. In a cosmological environment, Robertson–Walker metrics are conformally flat and this implies that conformally coupled particles, such as massless fermions and vector bosons, cannot be produced [4]<sup>1</sup>. On the other hand, particle masses break conformal invariance and serves as a source of particle creation. When the Hubble parameter  $H$  is much larger than the particle mass  $m$ , that is  $H \gg m$ , the number density of created particle is constant [1]

$$n = \frac{m^3}{24\pi^2}, \quad (1)$$

whereas for  $H \ll m$  particle creation is negligible and  $n$  decreases as  $1/a^3$ , where  $a$  is the cosmological scale factor, due to the expansion of the universe. For the sake of simplicity, in what follows we assume that, for  $H > m$ , the particle number density is given by eq. (1) and particle production stops instantaneously when  $H = m$ .

We notice here that formula (1) is valid for any power law expansion of the scale factor  $a(t) \propto t^q$  [1] – in standard cosmology, for matter or radiation dominated universe one has  $q = 2/3$  or  $q = 1/2$ , respectively, whereas braneworld cosmology enforces  $q = 1/3$  or  $q = 1/4$ . However, the approximation which led to (1) is found to be valid only within one order of magnitude, as different

epochs of the universe would result in different number densities today [6]. In particular, braneworld regimes tend to produce roughly 10 times more fermions than standard cosmologies. While we will work with the analytic expression (1) throughout the paper, we will present more realistic results when computing numerical values.

*Standard cosmology* – In standard cosmology, the phenomenon is usually negligible. The universe expansion rate during the radiation dominated epoch is

$$H_{SC} = \left( \frac{8\pi^3 g_*}{90} \right)^{1/2} \frac{T^2}{M_{Pl}}, \quad (2)$$

where  $g_*$  is the effective number of relativistic degrees of freedom and  $T$  the universe temperature. Gravitational production of particles with mass  $m_X$  stops when  $H_{SC} = m_X$  at the temperature (assuming it is below the reheating temperature, that is, the highest temperature at which thermodynamical equilibrium had been established)

$$T_{SC} \simeq 9 \cdot 10^9 \left( \frac{m_X}{100 \text{ GeV}} \right)^{1/2} \left( \frac{100}{g_*} \right)^{1/4} \text{ GeV}. \quad (3)$$

If the particle is stable, or if its lifetime is longer than the present age of the universe, its contribution to the total energy of the universe today would be

$$\Omega_X \simeq 2 \cdot 10^{-17} \left( \frac{m_X}{100 \text{ GeV}} \right)^{5/2}, \quad (4)$$

where a dilution factor of  $\sim 0.1$  has been taken into account.

*Braneworld cosmology* – In braneworld cosmology the picture can be much more interesting<sup>2</sup>. Focusing on the case with one extra dimension compactified on a circle, the effective four dimensional Friedman equation is [8]

$$H^2 = \frac{8\pi\rho}{3M_{Pl}^2} \left( 1 + \frac{\rho}{2\Lambda} \right), \quad (5)$$

<sup>1</sup> However, quantum conformal anomaly could circumvent this exclusion principle and allow for noticeable production of even massless gauge bosons [5].

<sup>2</sup> For instance, gravitational particle production in this framework has been considered in the context of inflation in [7].

where  $\rho$  is the energy density of ordinary matter on the brane,

$$\Lambda = \frac{48\pi M_*^6}{M_{Pl}^2} \quad (6)$$

is the brane tension and  $M_*$  the true gravity scale of the five dimensional theory. The transition temperature  $T_*$  is the temperature at which the evolution of the universe switches from braneworld regime to standard one and, if the universe is radiation dominated, it is

$$\begin{aligned} T_* &= 2 \left( \frac{180}{\pi g_*} \right)^{1/4} \left( \frac{M_*^3}{M_{Pl}} \right)^{1/2} \\ &\simeq 20 \left( \frac{100}{g_*} \right)^{1/4} \left( \frac{M_*}{10^5 \text{ GeV}} \right)^{3/2} \text{ MeV}. \end{aligned} \quad (7)$$

For  $T > T_*$ , the universe is in braneworld regime, and the expansion rate is faster than the standard one

$$H_{BC} = \frac{\pi^2 g_*}{180} \frac{T^4}{M_*^3}. \quad (8)$$

The freeze-out temperature of gravitational particle production is

$$\begin{aligned} T_{BC} &\simeq 1.2 \cdot 10^4 \left( \frac{100}{g_*} \right)^{1/4} \\ &\cdot \left( \frac{m_X}{100 \text{ GeV}} \right)^{1/4} \left( \frac{M_*}{10^5 \text{ GeV}} \right)^{3/4} \text{ GeV}. \end{aligned} \quad (9)$$

In order for the freeze-out temperature to be higher than  $T_*$ , but smaller than the five dimensional Planck mass, as required by consistency, the following relations between  $M_*$  and  $m_X$  must hold

$$0.2 m_X \lesssim M_* \lesssim 8 \cdot 10^{11} \left( \frac{m_X}{100 \text{ GeV}} \right)^{1/3} \text{ GeV}. \quad (10)$$

Hence, if these limits are satisfied, a period of braneworld gravitational particle production may<sup>3</sup> have taken place and, if the particle  $X$  is stable or quasi-stable,  $\Omega_X$  today would be

$$\Omega_X = 2 \left( \frac{m_X}{100 \text{ GeV}} \right)^{13/4} \left( \frac{10^5 \text{ GeV}}{M_*} \right)^{9/4}, \quad (11)$$

where as before a dilution factor of  $\sim 0.1$  has been accounted for.

*Phenomenology* – If the transition temperature  $T_*$  is smaller than the reheating temperature  $T_R$ , i.e. the universe went through a period of braneworld cosmology after inflation, gravitational particle production could have been very efficient. As it has been shown previously,

this inequality depends upon the five dimensional gravity mass scale, and one should verify that the freeze-out temperature for gravitational interaction is indeed higher than  $T_*$ , see eq. (10). On the other hand, if  $T_* > T_R$ , when the universe exited the inflationary period it started expanding following the standard Friedman equation and the existence of extra dimensions was essentially irrelevant, as long as towers of KK modes do not play any relevant rôle.

Let us now consider possible implications of this picture for the contemporary universe. If there existed a stable or quasi-stable weakly interacting or sterile (i.e. which interacts only gravitationally) particle  $X$ , it would contribute to the cosmological dark matter today. If we require  $\Omega_X = \Omega_{DM}$  and  $\Omega_{DM}^{obs} = 0.25$ , we find

$$\begin{aligned} M_* &\simeq 2 \cdot 10^5 \left( \frac{m_X}{100 \text{ GeV}} \right)^{13/9} \text{ GeV}, \\ T_* &\simeq 100 \left( \frac{m_X}{100 \text{ GeV}} \right)^{13/6} \text{ MeV}. \end{aligned} \quad (12)$$

It is noteworthy that, if such a particle were sterile, it would be essentially impossible to produce in collider experiments. Moreover, if that were the case, also the early universe would have had only gravitational production available, as the other mechanisms are cut off. Had we taken the numerical value for  $n_X$  [6] we would have obtained an extra factor of 3 for  $M_*$ , and  $T_*$  would have been five times higher. We are also neglecting possible (unknown) sources of entropy dilution at later times.

Interesting considerations arise if the particle  $X$  is the gravitino, the supersymmetric partner of the graviton. If it is the Lightest Supersymmetric Particle (LSP) and R-parity is conserved, it is stable and a good dark matter candidate; of course its energy density today must not overclose the universe. If it is not the LSP, it is unstable and its decay products had not to spoil the successful predictions of the Big Bang Nucleosynthesis (BBN) and overclose the universe. In standard cosmology, the gravitino is produced out of equilibrium after inflation by particle inelastic scattering, supersymmetric particle decay (the process is relevant only if the gravitino is the LSP) and possible model-dependent processes involving the inflaton or other scalar field (dilaton, moduli, etc.). In braneworld cosmology, the picture is a little different, because the universe expansion rate is faster and KK states can be excited as well, see e.g. Ref. [9]. Here gravitational particle production is taken into account.

Let us begin with the simplest picture, where we consider only the gravitino 0-mode and we neglect every other production mechanisms. If the gravitino is stable and we demand  $\Omega_{3/2} \leq \Omega_{DM}^{obs}$ , we obtain

$$M_* \gtrsim 2 \cdot 10^5 \left( \frac{m_{3/2}}{100 \text{ GeV}} \right)^{13/9} \text{ GeV}. \quad (13)$$

This is not a strong bound, because in models where the gravitino is the LSP it can be very light, even in the eV range. Moreover, since  $M_*$  is constrained to be

<sup>3</sup> The smallest  $T_{BC}/M_*$  ratio within these limits is about  $10^{-3}$ .

at least around  $10^4$  GeV from BBN, gravitationally produced gravitinos lighter than 100 GeV would be irrelevant today.

On the other hand, if the gravitino is unstable, its decay products can alter nuclei primordial abundances. At the BBN, the (diluted) gravitino number density to entropy density ratio would be

$$Y_{3/2} = 5 \cdot 10^{-11} \left( \frac{m_{3/2}}{100 \text{ GeV}} \right)^{9/4} \left( \frac{10^5 \text{ GeV}}{M_*} \right)^{9/4} \quad (14)$$

and, assuming that the main decay channel is hadronic, successful BBN requires [10]

$$Y_{3/2}^{\text{allowed}} \leq 10^{-16} \quad (15)$$

for  $m_{3/2} = 1$  TeV. This implies

$$M_* \gtrsim 3 \cdot 10^8 \text{ GeV}. \quad (16)$$

It is interesting to notice that the upper bound coming from the thermally produced gravitinos [11] is close to the lower bound obtained here. For instance, the same  $Y_{3/2}$  as in (15) gives

$$M_* \lesssim 2 \cdot 10^9 \text{ GeV}. \quad (17)$$

Notice further that using numerical estimates for the gravitationally produced gravitinos, as borrowed from [6], the lower limit will become extremely close to (17). Unfortunately other uncertainties affect the estimates just outlined, namely the details of the inflationary epoch, whether there has been further entropy release at late times, and so on. It is nevertheless remarkable how little window is left open for  $M_*$  in this scenario.

Thus, thermally and gravitationally produced gravitinos put competing limits on the fundamental mass scale of the theory, see fig. 1. This can be easily understood because gravitational production in braneworld becomes more efficient as the transition temperature drops (in this case the production stops later and there is little dilution afterwards), whereas the abundance of thermal gravitinos grows with it.

Of course this is the strongest limit, indeed if the gravitino were slightly lighter or heavier, or if the main channel were not hadronic, these constraints would be slightly relaxed [10].

As another possible application of the results obtained in this letter, we mention the rôle braneworld gravitational particle production may have in the generation of the baryon asymmetry of the universe. This mechanism allows for noticeable production of very weakly interacting particles which can later on decay out of equilibrium. In this case, a very low (TeV) gravity scale does not represent a problem, but is favorable for baryogenesis, as discussed in Ref. [12].

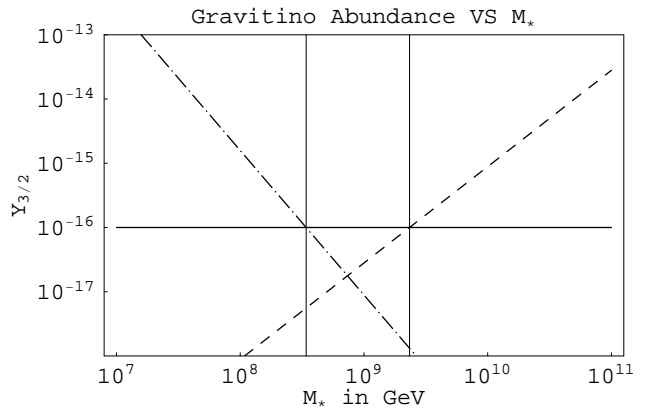


FIG. 1: Gravitino abundances as a function of  $M_*$ , for a  $m_X = 1$  TeV. Both gravitational (dot-dashed line) and thermal (dashed line) gravitinos are shown. The constraint (15) is the solid horizontal line, while the allowed range for  $M_*$  in this case lies in the band between the two vertical lines. Numerical results would move the lower limit closer to the upper one.

*Conclusion* – Particle production in time varying background metric of an expanding universe is a well known phenomenon which is usually only of theoretical interest in standard cosmology. Nevertheless, in theories with extra dimensions the expansion rate of the early universe could have been much faster, due to a modified Friedman equation. This fact translates in a very efficient mechanism of gravitational particle production, which has some intriguing phenomenological implications. We have shown how an abundance of dark matter compatible with observations can be easily produced, independently on the specific features of the dark matter candidate, once an order 100 GeV mass is given. This interesting feature relies on the fact that gravitational interactions could account for the necessary dark matter energy density, without the need for other mechanisms.

This fact holds when the LSP is the gravitino as well, provided that it is not too light, see eq. (13). On the other hand, an unstable heavy gravitino is dangerous for BBN. Thus, safeness for BBN would strongly constrain the extra-dimensional scale of gravity, as equation (16) shows. Furthermore, even more interestingly, this lower limit is in competition with the upper limit derived from thermal production of gravitinos. Once these two mechanisms are considered together there is little freedom in the choice of the parameters of the extra-dimensional model.

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- [1] S.G. Mamaev, V.M. Mostepanenko and A.A. Starobinsky, *ZhETF* **70**, 1577 (1976); *Sov. Phys. JETP* **43**, 823 (1976).
- [2] V.M. Frolov, S.G. Mamaev and V.M. Mostepanenko, *Phys. Lett. A* **55**, 389 (1976);  
A.A. Grib, S.G. Mamaev and V.M. Mostepanenko, *Gen. Rel. Grav.* **7**, 535 (1976);  
V.A. Kuzmin and I.I. Tkachev, *JETP Lett.* **68**, 271 (1998), [arXiv:hep-ph/9802304](#);  
D.J.H. Chung, E.W. Kolb and A. Riotto, *Phys. Rev. D* **59**, 023501 (1999), [arXiv:hep-ph/9802238](#);  
V.A. Kuzmin and I.I. Tkachev, *Phys. Rep.* **320**, 199 (1999), [arXiv:hep-ph/9903542](#).
- [3] A.D. Dolgov and D.P. Kirilova, *Yad. Fiz.* **51**, 273 (1990); *Sov. J. Nucl. Phys.* **51**, 172 (1990);  
J.H. Traschen and R.H. Brandenberger, *Phys. Rev. D* **42**, 2491 (1990);  
Y. Shtanov, J.H. Traschen and R.H. Brandenberger, *Phys. Rev. D* **51**, 5438 (1995), [arXiv:hep-ph/9407247](#);  
G.F. Giudice, M. Peloso, A. Riotto and I. Tkachev, *JHEP* **08** (1999) 014, [arXiv:hep-ph/9905242](#);  
L. Kofman, A.D. Linde and A.A. Starobinsky, *Phys. Rev. Lett.* **73**, 3195 (1994), [arXiv:hep-th/9405187](#).
- [4] L. Parker, *Phys. Rev. Lett.* **21**, 562 (1968).
- [5] A.D. Dolgov, *Pisma ZhETF* **32**, 673 (1980);  
A.D. Dolgov, *ZhETF* **81**, 417 (1981); *Sov. Phys. JETP* **54**, 223 (1981).
- [6] V. Kuzmin and I. Tkachev, *Phys. Rev. D* **59**, 123006 (1999), [arXiv:hep-ph/9809547](#).
- [7] E.J. Copeland, A.R. Liddle and J.E. Lidsey, *Phys. Rev. D* **64** 023509 (2001), [arXiv:astro-ph/0006421](#).
- [8] J.M. Cline, C. Grojean and G. Servant, *Phys. Rev. Lett.* **83**, 4245 (1999), [arXiv:hep-ph/9906523](#);  
P. Binétruy, C. Deffayet, U. Ellwanger and D. Langlois, *Phys. Lett. B* **477**, 285 (2000), [arXiv:hep-th/9910219](#).
- [9] C. Bambi and F. R. Urban, *JCAP* **0709**, 018 (2007), [arXiv:0705.4227 \[hep-ph\]](#).
- [10] M. Kawasaki, K. Kohri and T. Moroi, *Phys. Rev. D* **71**, 083502 (2005), [arXiv:astro-ph/0408426](#).
- [11] N. Okada and O. Seto, *Phys. Rev. D* **71**, 023517 (2005), [arXiv:hep-ph/0407235](#);
- [12] C. Bambi, A.D. Dolgov and K. Freese, *JCAP* **04** (2007) 005, [arXiv:hep-ph/0612018](#).